

### Exercise 3

Find the linearization  $L(x)$  of the function at  $a$ .

$$f(x) = \sqrt{x}, \quad a = 4$$

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#### Solution

Start by finding the corresponding  $y$ -value to  $x = 4$ .

$$f(4) = \sqrt{4} = 2$$

Then find the slope of the tangent line to the function at  $x = 4$  by computing  $f'(x)$ ,

$$\begin{aligned} f'(x) &= \frac{d}{dx} \sqrt{x} \\ &= \frac{1}{2} x^{-1/2} \\ &= \frac{1}{2\sqrt{x}}, \end{aligned}$$

and plugging in  $x = 4$ .

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

Now use the point-slope formula to obtain the equation of the line going through  $(4, 2)$  with slope  $1/4$ .

$$y - f(4) = f'(4)(x - 4)$$

$$y - 2 = \frac{1}{4}(x - 4)$$

$$y - 2 = \frac{1}{4}x - 1$$

$$y = \frac{1}{4}x + 1$$

Therefore, the linearization of the function  $f(x)$  at  $a = 4$  is

$$L(x) = \frac{1}{4}x + 1.$$

Below is a plot of the function and the linearization at  $a = 4$  versus  $x$ .

